DC Motor Speed Control for Electric Locomotive Equipped by Multi-level DC-DC Converter

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Abstract **–The problem of DC motor speed controller design for electric locomotive equipped by multi-level DC-DC converter is discussed where control system consists of two feedback loops. In the first one the armature current control for DC motor is provided by means of pulse-width modulated control for multi-level DC-DC converter. In the second one the DC motor speed control is maintained. Proportional-integral (PI) controllers are designed for armature current and motor speed based on singular perturbation technique such that multi-time-scale motions are artificially induced in the closed-loop system. Numerical simulations are included in order to show the efficacy of the proposed design methodology.**

Index Terms **–DC-DC motor speed control, converters, pulse-width modulation, PI controller, singular perturbation method.**

I. INTRODUCTION

INCREASE OF CAPACITY OF ELECTRIC locomotives conducts to an increase of losses in networks of a supply voltage. Modern electro-carts of a direct current are usually calculated for operation on networks with the supply voltage 1,5 kV and 3 kV. Progress of power semi-conductor switching devices makes possible to increase the supply voltage up to 12-18 kV on the basis of application of converters of the high-voltage direct voltage into the direct voltage corresponding to the level of traction electric motors [1-5]. Therefore an interest to systems of electric locomotive equipped by DC-DC converters has been renewed. Transition to contact networks with the raised supply voltage requires development of methods of controller design for engines of electric locomotives with the voltage converters.

This paper is a continuation of [4,5] where the multi-level DC-DC converter of a contact catenary 12kV in voltage 3kV for an electric locomotive has been offered. Then the mathematical model of this multi-level DC-DC converter as well the proportional-integral (PI) controller with an additional low-pass filtering were derived in [6]. The problem of controller design was reduced to the continuous-time controller design based on Filippov's average approach [7-9]. In this paper the problem of DC motor speed controller design for electric locomotive equipped by multi-level DC-DC converter [4,5] is discussed where control system consists of two feedback loops shown on Fig.1. In the first one the armature current control for DC motor is provided by means of pulse-width modulated control for multi-level DC-DC converter shown on Fig.2. In the second one the DC motor speed control is maintained. Both of controllers for armature current and motor speed are designed based on singular perturbation technique such that multi-time-scale motions are artificially induced in the closed-loop system [10].

The paper is organized as follows. First, general structure of the discussed DC motor speed control system is highlighted. Second, the mathematical model is presented for the multi-level DC-DC converter where its load is series connection of two DC motors of electric locomotive wheel pair. Third, the armature current controller and motor speed controller are designed based on singular perturbation technique. Finally, simulation results of the closed-loop control system are included in order to show the efficacy of the proposed design methodology.

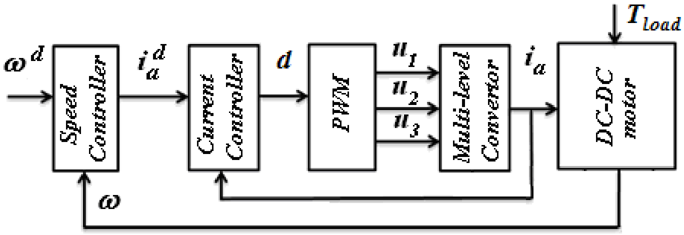

Fig. 1. Block diagram of control system

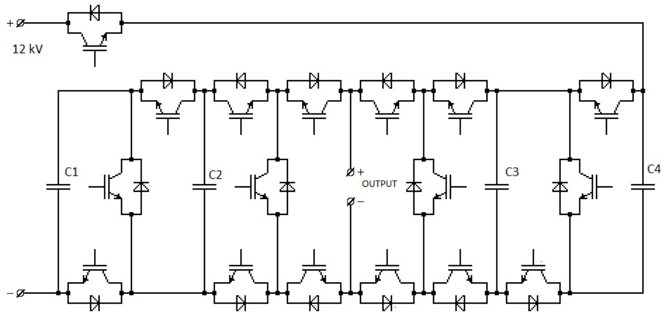

Fig. 2. The multi-level DC-DC converter circuit

II. CONTROL PROBLEM STATEMENT

The current controller is being designed so that to maintain the desired value of the DC-DC motor armature current i_a, that is

$$\lim_{t\to\infty} i_a(t) = i_a^d, \quad (1)$$

where i_a^d is the desired value (reference input) of the armature current. The speed controller is being designed so that to maintain the desired value of the DC-DC motor speed ω, that is

$$\lim_{t\to\infty} \omega(t) = \omega^d, \quad (2)$$

where ω^d is the desired value (reference input) of the DC-DC motor speed. Moreover, the controlled transients of ω and i_a should have the desired settling time without overshoot.

III. AVERAGED MODEL OF MULTI-LEVEL CONVERTER

The discussed multi-level converter operation consists of a periodical sequence of the three stages which can be defined by the switching functions u_1, u_2, and u_3 shown on Fig.3, that are:

Stage 1: $u_1 = 1,\ u_2 = 0,\ u_3 = 0.$

Stage 2: $u_1 = 0,\ u_2 = 1,\ u_3 = 0.$

Stage 3: $u_1 = 0,\ u_2 = 0,\ u_3 = 1.$

In order to avoid the effect of operation condition differences in the charge-discharge of capacitors, the switching sequence is doing as the following one: Stage 1, Stage 2, Stage 3, Stage 1, Stage 3, Stage 2, etc.

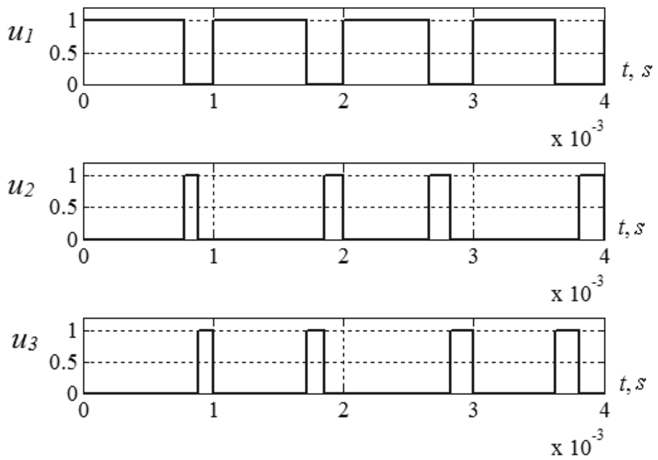

Fig. 3. Plots of the switching functions u_1, u_2, and u_3

A discontinuous control strategy is provided by the pulse-width modulator where the input signal of the modulator (duty ratio function) is defined as the scalar variable $d(t)$ which takes values in the interval $[0,1]$. The output signals of the modulator are defined as the switching functions u_1, u_2, and u_3 given by

$$u_1 = \begin{cases} 1 \ \ for \ \ t_\kappa < t \le t_\kappa + d(t_\kappa)T_s \\ 0 \ \ for \ \ t_\kappa + d(t_\kappa)T_s < t \le t_\kappa + T_s \end{cases}$$

$$u_2 = \begin{cases} 0 \ \ for \ \ t_\kappa < t \le t_\kappa + d(t_\kappa)T_s \\ 1 \ \ for \ \ t_\kappa + d(t_\kappa)T_s < t \le t_\kappa + [1-d(t_\kappa)]T_s/2 \\ 0 \ \ for \ \ t_\kappa + [1-d(t_\kappa)]T_s/2 < t \le t_\kappa + T_s \end{cases}$$

$$u_3 = \begin{cases} 0 \ \ for \ \ t_\kappa < t \le t_\kappa + d(t_\kappa)T_s \\ 0 \ \ for \ \ t_\kappa + d(t_\kappa)T_s < t \le t_\kappa + [1-d(t_\kappa)]T_s/2 \\ 1 \ \ for \ \ t_\kappa + [1-d(t_\kappa)]T_s/2 < t \le t_\kappa + T_s \end{cases}$$

where T_s is the PWM sampling period, $d(t_\kappa)$ is the value of the duty ratio function when $t = t_\kappa$, $t_\kappa = \kappa T_s$, and $\kappa = 0,1,2,\ldots$.

Consider the model of DC motor equipped by multi-level DC-DC converter with the ideal switches in the owing form [6]:

$$\frac{di_a}{dt} = -\frac{R_a}{L_a} i_{\text{я}} - \frac{E_a}{L_a} + \frac{u_{C1}}{L_a} u_2 + \frac{u_{C3}}{L_a} u_3,$$

$$\frac{du_{C1}}{dt} = \frac{1}{R_{in}C_1}(E_1 - \Sigma u_{Ci})u_1 - \frac{i_a}{C_1 + C_2} u_2,$$

$$\frac{du_{C2}}{dt} = \frac{1}{R_{in}C_2}(E_1 - \Sigma u_{Ci})u_1 - \frac{i_a}{C_1 + C_2} u_2, \qquad (3)$$

$$\frac{du_{C3}}{dt} = \frac{1}{R_{in}C_3}(E_1 - \Sigma u_{Ci})u_1 - \frac{i_a}{C_3 + C_4} u_3,$$

$$\frac{du_{C4}}{dt} = \frac{1}{R_{in}C_4}(E_1 - \Sigma u_{Ci})u_1 - \frac{i_a}{C_3 + C_4} u_3,$$

$$\frac{d\omega}{dt} = \frac{k_2}{J} i_a - \frac{T_{load}}{J},$$

where E_1 is the supply voltage of a power line, $E_1 = 12$ kV, R_{in} is the active resistance of catenary, u_{Ci} are capacitor voltages, T_{load} is the normalized load torque, J is the normalized inertia torque, k_1 and k_2 are the motor form factors. The load consists of series connection of two identical DC motors of the electric locomotive wheel pair with the total inductance L_a, the total active resistance R_a, and the total reverse electromotive force (back EMF) E_a in the armature circuit.

Assumption 1: The pulse-width modulator is not saturated, that is the following inequality $0 < d < 1$ holds.

Assumption 2: The sampling period T_s is assumed to be sufficiently small in compare with time constants associated with the dynamics of the converter.

From Assumptions 1 and 2, by following to the Filippov's approach [9], the geometric approach to PWM control [7,8], and by taking into account the definition of the switching functions u_1, u_2, and u_3, the response of discontinuously controlled system given by (3) coincides with Filippov's average model

$$\frac{dI_a}{dt} = -\frac{R_a}{L_a} I_a - \frac{E_a}{L_a} + \frac{U_{C1} + U_{C3}}{2L_a} - \frac{U_{C1} + U_{C3}}{2L_a} d,$$

$$\frac{dU_{C1}}{dt} = -\frac{I_a}{2(C_1 + C_2)} + \left[\frac{E_1 - \Sigma U_{Ci}}{R_{in}C_1} + \frac{I_a}{2(C_1 + C_2)}\right] d,$$

$$\frac{dU_{C2}}{dt} = -\frac{I_a}{2(C_1 + C_2)} + \left[\frac{E_1 - \Sigma U_{Ci}}{R_{in}C_2} + \frac{I_a}{2(C_1 + C_2)}\right] d, \qquad (4)$$

$$\frac{dU_{C3}}{dt} = -\frac{I_a}{2(C_3 + C_4)} + \left[\frac{E_1 - \Sigma U_{Ci}}{R_{in}C_3} + \frac{I_a}{2(C_3 + C_4)}\right] d,$$

$$\frac{dU_{C4}}{dt} = -\frac{I_a}{2(C_3 + C_4)} + \left[\frac{E_1 - \Sigma U_{Ci}}{R_{in}C_4} + \frac{I_a}{2(C_3 + C_4)}\right] d,$$

$$\frac{d\omega}{dt} = \frac{k_2}{J} I_a - \frac{T_{load}}{J},$$

where the variables $I_a, U_{C1}, U_{C2}, U_{C3}, U_{C4}$ are the averaged values of variables $i_a, u_{C1}, u_{C2}, u_{C3}, u_{C4}$ and $\Sigma U_{Ci} = U_{C1} + U_{C2} + U_{C3} + U_{C4}$.

Take $C_1 = C_2 = C_3 = C_4 = C$ and, under assumption that the conditions $U_C = U_{C1} = U_{C2} = U_{C3} = U_{C4}$ hold, instead of the full order averaged system (4), the reduced order averaged system can by considered, that is

$$\frac{dI_a}{dt} = -\frac{R_a}{L_a} I_a - \frac{E_a}{L_a} + \frac{U_C}{L_a} - \frac{U_C}{L_a} d,$$

$$\frac{dU_C}{dt} = -\frac{I_a}{4C} + \left[\frac{E_1 - 4U_C}{R_{in}C} + \frac{I_a}{4C}\right] d, \qquad (5)$$

$$\frac{d\omega}{dt} = \frac{k_2}{J} I_a - \frac{T_{load}}{J}.$$

The analysis of the eigenvalue spectrum of (5), in case when $d = const$, reveal the stability and two-time scale nature of transients in the system (5), where I_a and ω U_C are the slow variables, U_C is the fast variable. Hence, the further order reduction of the averaged system (5) can be done by assumption that the condition $dU_C / dt = 0$ holds. Moreover, the sake of simplicity, by the neglecting of small variations for U_C, let the condition $U_C \approx E_1 / 4$ holds. Consider the total reverse electromotive force (back EMF) E_a in the armature circuit as given by

$$E_a = k_1 \omega . \quad (6)$$

Finally, from (5) and (6), on purpose of controller design, the following averaged model of DC motor equipped by multi-level DC-DC converter will be treated:

$$\begin{aligned} \frac{dI_a}{dt} &= -\frac{R_a}{L_a} I_a - \frac{k_1}{L_a}\omega + \frac{E_1}{4L_a} - \frac{E_1}{4L_a} d, \\ \frac{d\omega}{dt} &= \frac{k_2}{J} I_a - \frac{T_{load}}{J}. \end{aligned} \quad (7)$$

IV. ARMATURE CURRENT CONTROLLER

The current regulator is being designed for the discussed multi-level converter so that to maintain the desired value of the armature current i_a that is (1). Consider the current continuous-time controller given by the following differential equation:

$$\mu_a^2 d^{(2)} + d_a \mu_a d^{(1)} = k_a [(i_a^d - i_a) / T_a - i_a^{(1)}] \quad (8)$$

where μ_a is a small positive parameter of the controller, $\mu_a > 0,\ d_a > 0$, and $T_a > 0$. The control law (8) can be expressed in terms of the Laplace transform that is the structure of the PI controller with an additional low-pass filtering given by

$$d(s) = \frac{k_a}{\mu_a(\mu_a s + d_a)} \left\{ \frac{1}{sT_a}[i_a^d(s) - i_a(s)] - i_a(s) \right\}.$$

The closed-loop system analysis is provided below based on the consideration of the reduced average system (7) with controller (8) where the current i_a is replaced by I_a, that is

$$\begin{aligned} I_a^{(1)} &= -\frac{R_a}{L_a} I_a - \frac{k_1}{L_a}\omega + \frac{E_1}{4L_a} - \frac{E_1}{4L_a} d, \\ \omega^{(1)} &= \frac{k_2}{J} I_a - \frac{T_{load}}{J}, \\ \mu_a^2 d^{(2)} + d_a \mu_a d^{(1)} &= k_a \left[\frac{i_a^d - I_a}{T_a} - I_a^{(1)} \right]. \end{aligned} \quad (9)$$

Denote $d_1 = d,\ d_2 = \mu_a d^{(1)}$. The replacement of $I_a^{(1)}$ in the third equation of (9) by the right member of the first equation of (9) yields the reduced order averaged closed-loop system in the form

$$\begin{aligned} I_a^{(1)} &= -\frac{R_a}{L_a} I_a - \frac{k_1}{L_a}\omega + \frac{E_1}{4L_a} - \frac{E_1}{4L_a} d, \\ \omega^{(1)} &= \frac{k_2}{J} I_a - \frac{T_{load}}{J}, \\ \mu_a d_1^{(1)} &= d_2, \\ \mu_a d_2^{(1)} &= \frac{k_a E_1}{4L_a} d_1 - d_a d_2 \\ &\quad + k_a \left[\frac{i_a^d - I_a}{T_a} + \frac{R_a}{L_a} I_a + \frac{k_1}{L_a}\omega - \frac{E_1}{4L_a} \right]. \end{aligned} \quad (10)$$

Since μ_a is the small parameter, the above equations (10) are the singularly perturbed differential equations [11-15]. Hence, fast and slow modes are artificially forced in the closed-loop system (10) as $\mu_a \to 0$. The degree of time-scale separation between these modes depends on the parameter μ_a. From (10), the averaged fast-motion subsystem (FMS)

$$\begin{aligned} \mu_a d_1^{(1)} &= d_2, \\ \mu_a d_2^{(1)} &= \frac{k_a E_1}{4L_a} d_1 - d_a d_2 \\ &\quad + k_a \left[\frac{i_a^d - I_a}{T_a} + \frac{R_a}{L_a} I_a + \frac{k_1}{L_a}\omega - \frac{E_1}{4L_a} \right] \end{aligned} \quad (11)$$

results, where I_a and ω are treated as the frozen variables during the transients in (11).

Remark 1: The stability of FMS transients of (11) is provided by selection of the gain k_a such that the condition $k_a E_1 < 0$ holds given that $\mu_a > 0$ and $d_a > 0$.

Assume that the control law parameters k_a, μ_a, and d_a have been selected such that the FMS (11) is stable as well as time-scale decomposition is maintained in the closed-loop system (10). Take, for example,

$$k_a = -\frac{4L_a}{E_1},$$

then the FMS (11) characteristic polynomial is given by

$$\mu_a^2 s^2 + d_a \mu_a s + 1. \quad (12)$$

Letting $\mu_a \to 0$ in (11), we obtain the steady state (more precisely, quasi-steady state) of the FMS (11), where $d_1 = d_1^{id}$, that is the inverse dynamics solution, and

$$d_1^{id} = -\frac{4L_a}{E_1} \left[\frac{i_a^d - I_a}{T_a} + \frac{R_a}{L_a} I_a + \frac{k_1}{L_a}\omega - \frac{E_1}{4L_a} \right].$$

Substitution of $d_1 = d_1^{id}$ into the first equation of (10) yields the averaged slow-motion subsystem (SMS) given by

$$\begin{aligned} \frac{dI_a}{dt} &= \frac{i_a^d - I_a}{T_a}, \\ \frac{d\omega}{dt} &= \frac{k_2}{J} I_a - \frac{T_{load}}{J}. \end{aligned} \quad (13)$$

So, the average behavior of the current I_a is prescribed by the stable reference equation, that is the first equation in system (13), and by that the requirement (1) is maintained, in the average sense, that is

$$\lim_{t \to \infty} I_a(t) = i_a^d .$$

Remark 2: The parameter T_a is selected in accordance with the desired settling time t_{SMS} for the armature current I_a such that $T_a \approx t_{SMS} / 3$. The time-scale decomposition is maintained in the system (10) by selection of the parameter μ_a such that the condition $\mu_a \approx T_a / \eta_a$ holds, where η_a is the degree of time-scale separation between fast and slow modes. The desired damping of the FMS transients is provided by selection of the parameter d_a.

Finally, by taken into account that the transients of the armature current I_a in system (13) are much faster than the transients of the speed ω, take $I_a = i_a^d$, and on purpose of motor speed controller design, the following reduced model of DC motor will be treated:

$$\frac{d\omega}{dt} = \frac{k_2}{J} i_a^d - \frac{T_{load}}{J}, \tag{14}$$

where i_a^d is considered as the new control variable of the motor speed feedback loop.

V. DC MOTOR SPEED CONTROLLER

The DC motor speed controller is being designed so that to maintain the desired value of the motor speed ω, that is (2). Consider the motor speed continuous-time controller given by the following differential equation:

$$\mu_\omega \frac{di_a^d}{dt} = k_\omega \left[\frac{\omega^d - \omega}{T_\omega} - \frac{d\omega}{dt} \right] \tag{15}$$

where μ_ω is a small positive parameter of the controller, $\mu_\omega > 0,\ T_\omega > 0$. The control law (15) can be expressed in terms of the Laplace transform that is the structure of the conventional PI controller given by

$$i^d(s) = \frac{k_\omega}{\mu_\omega} \left\{ \frac{1}{sT_\omega} [\omega^d(s) - \omega(s)] - \omega(s) \right\}.$$

The closed-loop system analysis is provided below based on the consideration of the reduced model (14) with controller (15). The replacement of $d\omega / dt$ in (15) by the right member of (14) yields the closed-loop system in the form

$$\frac{d\omega}{dt} = \frac{k_2}{J} i_a^d - \frac{T_{load}}{J},$$

$$\mu_\omega \frac{di_a^d}{dt} = -\frac{k_\omega k_2}{J} i_a^d + k_\omega \left[\frac{\omega^d - \omega}{T_\omega} + \frac{T_{load}}{J} \right]. \tag{16}$$

The above equations (16) are the singularly perturbed differential equations where fast and slow modes are artificially forced as $\mu_\omega \to 0$ [11-15]. The degree of time-scale separation between these modes depends on the parameter μ_ω. From (16), the fast-motion subsystem (FMS)

$$\mu_\omega \frac{di_a^d}{dt} = -\frac{k_\omega k_2}{J} i_a^d + k_\omega \left[\frac{\omega^d - \omega}{T_\omega} + \frac{T_{load}}{J} \right] \tag{17}$$

results, where ω is treated as the frozen variable during the transients in (17).

Assume that the control law parameter k_ω has been selected such that $k_\omega = J / k_2$ then the FMS (17) characteristic polynomial is given by

$$\mu_\omega s + 1. \tag{18}$$

Then the FMS (17) is stable as well as time-scale decomposition is maintained in (16) by selection of μ_ω. Letting $\mu_\omega \to 0$ in (17), we obtain the steady state (more precisely, quasi-steady state) of the FMS (17), where $i_a^d = (i_a^d)^{id}$, that is the inverse dynamics solution, and

$$(i_a^d)^{id} = \frac{J}{k_2} \left[\frac{\omega^d - \omega}{T_\omega} + \frac{T_{load}}{J} \right].$$

Substitution of $i_a^d = (i_a^d)^{id}$ into the first equation of (16) yields the averaged slow-motion subsystem (SMS) given by

$$\frac{d\omega}{dt} = \frac{\omega^d - \omega}{T_\omega}. \tag{19}$$

Hence, after damping of fast transients of (17), we get from (16) the slow-moion subsystem (19). So, the behavior of the motor speed ω is prescribed by the stable reference equation (19) and by that the requirement (2) is maintained.

Note, time-scale decomposition between the both control loops is maintained by selection of controller parameters such that the conditions $\mu_a << T_a << \mu_\omega << T_\omega$ hold.

VI. SIMULATION OF CLOSED-LOOP SYSTEM

Let the DC motor and the multi-level converter parameters are as the following ones: $J = 150$ kG$\cdot$m^2, $L = 0.003\ H$, $C_1 = C_2 = C_3 = C_4 = 0.002\ F$, $R_a = 0.34\ \Omega$, $R_{in} = 0.1\ \Omega$, $E_1 = 12\,kV$. The sampling period of the pulse-width modulator is selected as $T_s = 0.001$ s. In accordance with the presented above design methodology, the following controller parameters were selected: $k_a = -4L_a / E_1 = -1 \cdot 10^{-6}$, $k_\omega = J / k_2 = 5.44$, $d_a = 2$, $\mu_a = 0.0013$ s, $T_a = 0.01$ s, $\mu_\omega = 0.1$ s, $T_\omega = 1$ s,

The simulation of the discussed DC motor equipped by multi-level converter based on the model (3) with controllers (8) and (15) has been done based on Matlab/Simulink Tools. The results of simulation are displayed in Figs.4-7, where the simulation results confirm the analytical calculations.

The deviations of ω and i_a on Figs.4-5 in the time instance equals 7s are exited by the step change of T_{load} from 9000 N$\cdot$m to 12000 N$\cdot$m. The deviation of ω on Fig.6 in the time instance equals 10 s is exited by the step change of E_1 from 12 kV to 11 kV (see Fig.7).

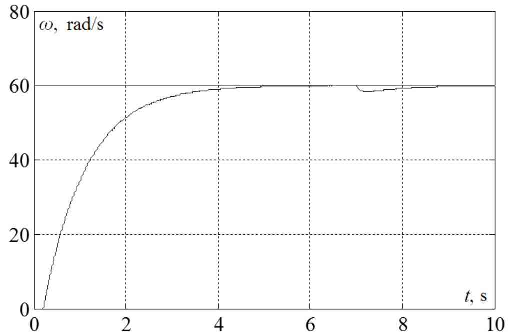

Fig. 4 Simulation results: Plot of ω (rad/s)

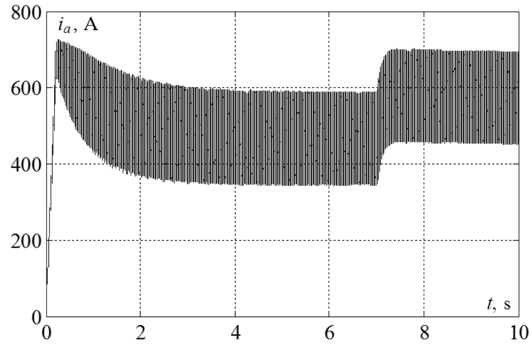

Fig. 5 Simulation results: Plots of i_a (A)

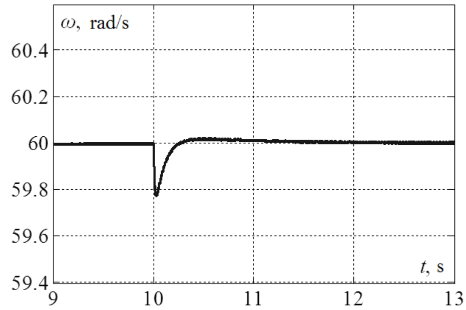

Fig. 6 Simulation results: Plot of ω (rad/s)

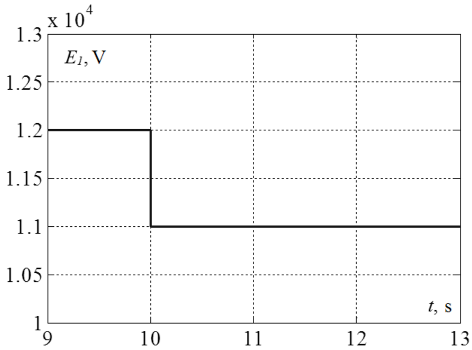

Fig. 7 Simulation results: Plot of E_1 (V)

VII. CONCLUSION

The advantage of the presented singular perturbation technique of controller design for the discussed DC motor equipped by multi-level DC-DC converter is that the desired transients of the motor speed are maintained in the presence of uncertainties of the supply voltage E_1 of a power line and load torque T_{load}. The other advantage is that analytical expressions for selection of controller parameters are derived, where controller parameters depend explicitly on the specifications of the desired behavior of motor speed.

VIII. ACKNOWLEDGEMENT

This work was supported by the government contract under grant no. 13.G36.31.0010 beginning to 22.10.2010 year.

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