

An approach to 3D-inversion of the time-domain marine electrical prospecting data

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SUMMARY

This paper is devoted to the methods and software of the 3D-inversion of the marine electromagnetic soundings data with a source in the form of a horizontal electrical line. The proposed method for the 3D-inversion is based on a direct determination of the geological objects parameters: their boundaries and electrical resistivity values. The computational experiment on the synthetic data shows a high accuracy of the method when it is applied to the three-dimensional marine electrical prospecting inverse problem.

Keywords: 3D-inversion, marine electrical prospecting, finite element method.

INTRODUCTION

The approaches to solving three-dimensional inverse problems are mainly developed on the following basis. A sample volume of the Earth (located under the acquisition system) is divided into the cells, and electrical conductivity and/or chargeability (depending on the acquisition technology) values are restored in each cell.

One of the most computationally efficient approaches to recover the electrophysical parameters in the cellular structures is actually based on the use of the same grid for solving the direct and inverse problems. It allows reducing the computational cost of a cell influence field calculation without switching to the simplified mathematical models when the "cellular" approach is used (Oldenburg et al. 2013).

However, the approach used for the electrophysical parameters recovery in the cellular structures may have the following disadvantages. As the number of cells is increased, the range of equivalency grows significantly, and the further development in this direction relates to the choice of the suitable regularization methods which allow "smoothing" the resulting distribution of the electrophysical parameters in the geological medium. As a result, if the regularization is not strong enough, the distribution pattern of the sought parameter becomes too "motley" (actually non-physical). With the regularization enhanced, the size and morphology of objects can still be determined very poorly, because the distribution pattern is heavily blurred.

In this paper, we propose an alternative approach to the 3D-inversion. The approach is based on a direct determination of the geological objects parameters: their boundaries and electrical resistivity (not on their extraction as a set of cells

from some quite smoothed distribution of one of the geophysical parameters). The proposed approach leads to the sharp reduction in the number of required parameters of the geoelectrical model without compromising its adequacy to the real medium.

MATHEMATICAL APPARATUS

Parameters of the three-dimensional geoelectrical models will be determined on the basis of minimizing the functional

$$\Phi(\mathbf{b}) = \sum_{l=1}^L \sum_{k=1}^K (v_{lk} \delta \mathcal{E}_{lk}(\mathbf{b}))^2 + \sum_{m=1}^M \alpha_m (\Delta b_m)^2 + \sum_{m=1}^M \gamma_m \sum_{s \in I_m} (\beta_m (b_m^0 - b_s^0) + \Delta b_m - \Delta b_s)^2, \quad (1)$$

where the first term is a functional error (or residual), and the other two are the regularizing additives.

Notation in formula (1) is the following.

$\delta \mathcal{E}_{lk} = \tilde{\mathcal{E}}_{lk} - \mathcal{E}_{lk}$ is an error (residual) in the signals, \mathcal{E}_{lk} is a signal registered in l -th

receiver at the time t_k , $\tilde{\mathcal{E}}_{lk}$ is the corresponding theoretical signal, that was obtained by solving the forward three-dimensional problem, \mathbf{b} is a vector of unknown parameters b_m , \mathbf{b}^0 is a vector of parameters b_m^0 that were obtained from the previous iteration of the nonlinear inversion procedure, $\Delta b_m = b_m - b_m^0$, α_m , β_m , and γ_m are the regularization parameters, I_m is a set of indexes of parameters used for smoothing with m -th parameter, v_{lk} is some weight reflecting the level of error in the signal in l -th receiver and

scale of the change of the received signal on time. Vector of the desired parameters \mathbf{b} includes conductivity values of required objects, as well as varying coordinates of the boundaries of these objects.

Minimization procedure of the functional (1) is based on the linearization of the theoretical signals $\tilde{\mathcal{E}}_{lk}$ by the parameters b_m in the neighborhood of b_m^0 . As a result, the deviation $\delta\mathcal{E}_{lk}$ between the theoretical $\tilde{\mathcal{E}}_{lk}$ and practical \mathcal{E}_{lk} signals is represented as

$$\delta\mathcal{E}_{lk}(\mathbf{b}) \approx \delta\mathcal{E}_{lk}(\mathbf{b}^0) + \sum_{m=1}^M \frac{\partial(\delta\mathcal{E}_{lk})}{\partial b_m} \Delta b_m, \quad (2)$$

where $\frac{\partial(\delta\mathcal{E}_{lk})}{\partial b_m}$ is derivative, reflecting the influence of change of the m -th parameter in l -th receiver at k -th time layer.

As a result, minimizing functional (1) takes the form:

$$\begin{aligned} \Phi(\mathbf{b}) = & \sum_{l=1}^L \sum_{k=1}^K \left(v_{lk} \delta\mathcal{E}_{lk}(\mathbf{b}^0) + v_{lk} \sum_{m=1}^M \frac{\partial(\delta\mathcal{E}_{lk})}{\partial b_m} \Delta b_m \right)^2 + \\ & + \sum_{m=1}^M \alpha_m (\Delta b_m)^2 + \\ & + \sum_{m=1}^M \gamma_m \sum_{s \in I_m} \left(\beta_m (b_m^0 - b_s^0) + \Delta b_m - \Delta b_s \right)^2. \end{aligned} \quad (3)$$

Effectiveness of the solution of the inverse problem is mainly determined by the methods and special procedures and weights of choice of regularization parameters α_m , β_m , γ_m .

Computational cost for minimizing the functional **Ошибка! Источник ссылки не найден.** is almost completely determined by the cost of the three-dimensional solution of forward problems in

the calculation of derivatives $\frac{\partial(\delta\mathcal{E}_{lk})}{\partial b_m}$ and theoretical values $\tilde{\mathcal{E}}_{lk}(\mathbf{b})$ of the signals in the receivers.

Solution of the forward problem for the calculation of $\tilde{\mathcal{E}}_{lk}(\mathbf{b})$ for time-domain techniques is performed using vector finite element method for a mathematical model that is based on the so-called

technology of field separation (Persova et al. 2011). This model has the form

$$\text{rot} \left(\frac{1}{\mu_0} \text{rot} \bar{\mathbf{A}}^a \right) + \sigma \frac{\partial \bar{\mathbf{A}}^a}{\partial t} = (\sigma - \sigma^n) \bar{\mathbf{E}}^n,$$

where $\bar{\mathbf{A}}^a = (A_x^a, A_y^a, A_z^a)$ is a vector potential of the anomalous field (defined by three-dimensional inhomogeneities), μ_0 is permeability of free space, σ^n is conductivity of host horizontally layered medium, $\bar{\mathbf{E}}^n$ is the electric field vector in a horizontally layered medium (normal field), and σ is the conductivity function of the three-dimensional medium.

The mathematical model for calculating the field that describes the impact of a 3D object has the following form

$$\text{rot} \left(\frac{1}{\mu_0} \text{rot} \bar{\mathbf{A}}^a \right) + \sigma \frac{\partial \bar{\mathbf{A}}^a}{\partial t} = (\sigma - \sigma^{3D-0}) \bar{\mathbf{E}}^{3D-0}, \quad (4)$$

where σ^{3D-0} and $\bar{\mathbf{E}}^{3D-0}$ are the distributions of conductivity and intensity of the electric field in the 3D medium, the field for which was calculated at the previous iteration of nonlinear inversion.

To find the normal components of the electromagnetic fields ($\bar{\mathbf{E}}^n$), the mathematical models are used that require much smaller (two orders or more) computational cost compared to three-dimensional solution of the problem without the normal field separation (Persova et al. 2011). When calculating the anomalous fields by eliminating a mesh refinement in the vicinity of the source, the computational costs is significantly (by 1-2 orders) lower than costs for solving the full three-dimensional problem.

RESULTS

The developed methods and the software have been tested on 3D-inversions of the marine electrical prospecting synthetic data. To carry out the computational experiment, a geoelectrical model that is specific for oil and gas deposits in the shelf zone has been formed.

This model is a five-layer horizontally layered medium that includes 3D-objects with high and low electrical resistivity, the objects that simulate the geological disturbances on the top (near the seabed) part of the section, 3D-objects with high electrical resistivity in the middle part of the section (on a depth of about 1,500 m) that simulate oil and gas deposits (these objects are the targets and are

of greatest interest during the electrical prospecting research), 3D-objects with low electrical resistivity in the middle part of the section that simulate the resistance decrease in the reservoir, and the high resistance deep 3D-objects that simulate the foundation uplift. A volume image of the 3D-objects of the "true" geoelectrical model and the plan view (the upper objects enclose the lower objects located strictly underneath or parts of them) and in section (the side view) are shown in Figure 1.

Practical signals for this model were synthesized using the 3D-modeling for 198 positions of the receiving-generator set that is moved along the sea surface and composed of a horizontal electric source line and of the 36 receiving lines arranged in three parallel rows.

The 3D-inversion was carried out in two stages. At the first stage, the volume of the investigated medium was divided into cells (the size of them increases with increasing the depth). And a preliminary distribution of resistivity was found fast enough. On the basis of this distribution, a starting geoelectric model shown in Figure 2 was made. At the second stage of the 3D-inversion, the parameters of the selected three-dimensional inhomogeneities were restored. Both their conductivities and geometrical characteristics (coordinates of their boundaries) of objects were among these parameters. Figures 3 and 4 show the geoelectrical models that were obtained at the intermediate and the final stages of the 3D-inversion.

The results show that the parameters (resistivity, position of the boundaries in plan and in depth) of the high resistance 3D-objects in the middle part of the section that simulates targets (oil and gas deposits) are determined with very high accuracy. The coordinates of the south-east target object, which consists of two components, were determined with an accuracy of about 100 m (that does not exceed the permitted step for a given structural mesh for moving boundaries).

The same error in depth (up to the top edge) does not exceed 1.5 % (which is considered as a very high degree of accuracy for the depths of about 1500 m). As for the target 3D-object on the west, its main components (located closer to the north) have also been found with very high precision. But the same error was higher on the south due to other objects parameters insufficient recovery, especially for the objects of the upper part of the section. The error in depth for this target object does not exceed 3% that is also a very good result for the given depth range.

We also note that the foundation structure (depth of

the upper edge) was matched quite well with the structure of the true model, except that the resistivity in the component parts of the 3D-object was found lower. However, firstly, the object is not a target, and secondly, for the better recovery of its properties the distance between the receiving lines should be increased.

CONCLUSION

The proposed approach is based on the direct search of the boundaries and electrophysical parameters of geological objects. It allows determining a depth, position and even a morphology of target objects with a very high accuracy, even in the complex geoelectric conditions where the target objects of a complex shape are necessary to allocate on the background of other geological structures.

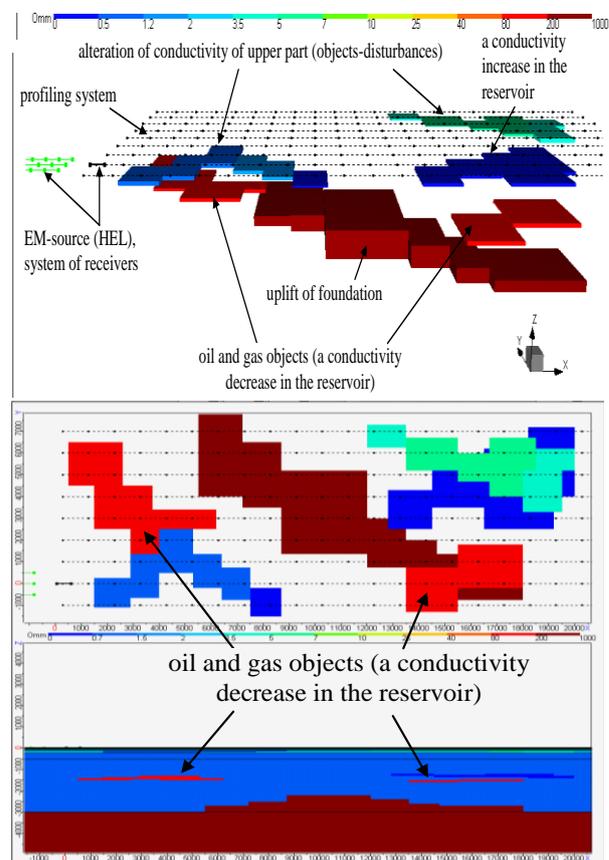


Figure 1. The "true" model

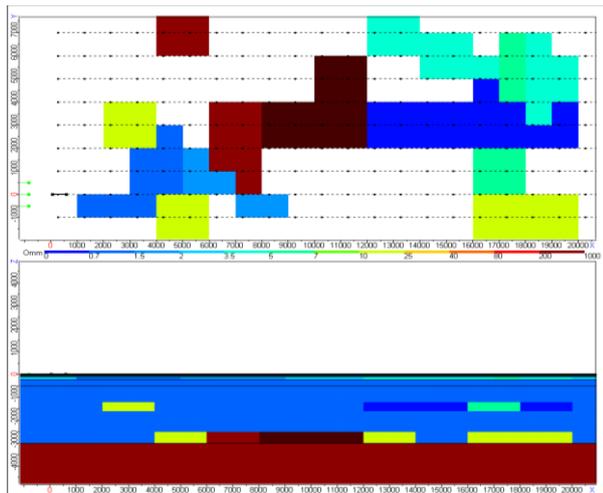
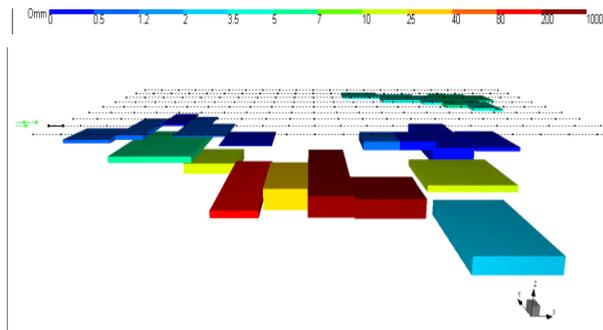


Figure 2. The starting model

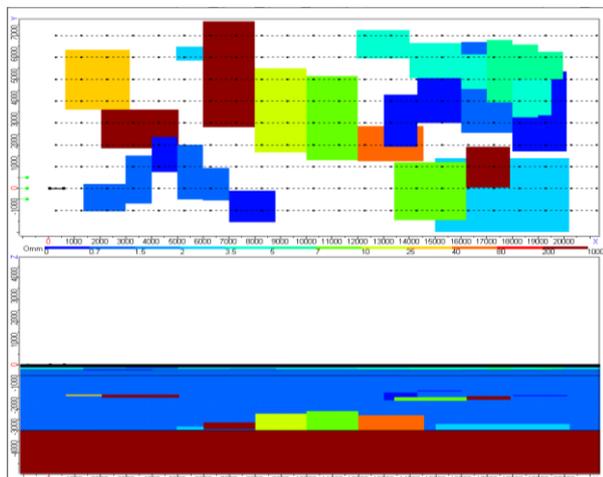
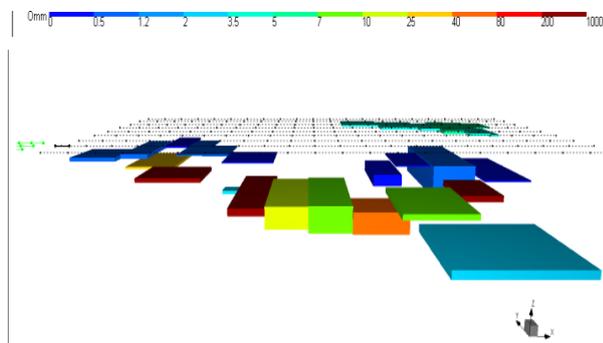


Figure 3. The model obtained on the intermediate stage of the 3D-inversion

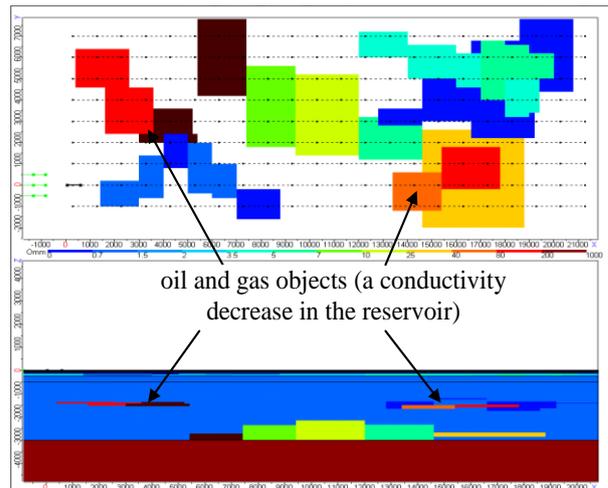
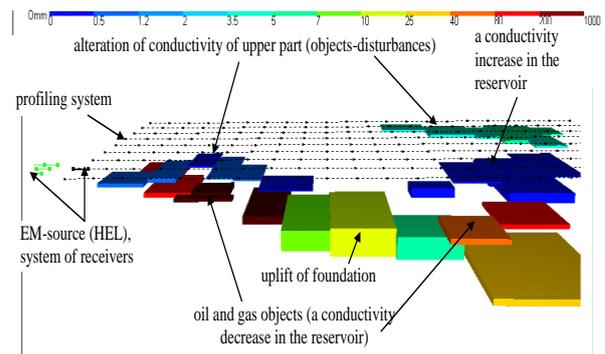


Figure 4. The final model obtained as a result of the 3D-inversion

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