ВГЗ N 6 8.12.

Волновая ф-я.

Ψ - компл. - я

$\Psi\Psi^* = |\Psi|^2 = \frac{dw}{dV}$, $dw = |\Psi|^2 dV = |\Psi|^2 dx\,dy\,dz$

$\int_0^\infty |\Psi|^2 dV = 1$ нормировка

суперпозиция $\Psi_1, \Psi_2, \dots \Psi_n$ $\Psi = \sum C_i \Psi_i$

C_i

Шрёдингер.

$-\frac{\hbar^2}{2m}\nabla^2\Psi + U(x,y,z,t)\Psi = i\hbar\frac{\partial\Psi}{\partial t}$ (*) $\hbar = \frac{h}{2\pi}$

$\nabla^2 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$, U, i

$$e^{-i\left(\frac{E}{\hbar}\right)t} \qquad \psi(x,y,z,t)=\psi'(x,y,z)\,e^{-i\left(\frac{E}{\hbar}\right)t}$$

$$-\frac{\hbar^2}{2m}\nabla^2\psi' + U(x,y,z)\,\psi' = E\psi' \qquad E\text{ - пол. эн-я}$$

$$\boxed{\nabla^2\psi' + \frac{2m}{\hbar^2}(E-U)\,\psi' = 0} \quad (*) \qquad \ddot{x}+\omega_0^2 x = 0$$

детерминизм

$U = f(x)$

$U(x) = 0$, $E = W_k$, по OX

$$\frac{\partial^2 \psi'}{\partial x^2} + \frac{2m}{\hbar^2} E \psi' = 0 \Leftrightarrow \frac{\partial^2 \psi'}{\partial x^2} + k^2 \psi' = 0$$

$$k^2 = \frac{2mE}{\hbar^2} , \quad \psi'(x) = A e^{ikx} , \quad E = \frac{\hbar^2 k^2}{2m}$$

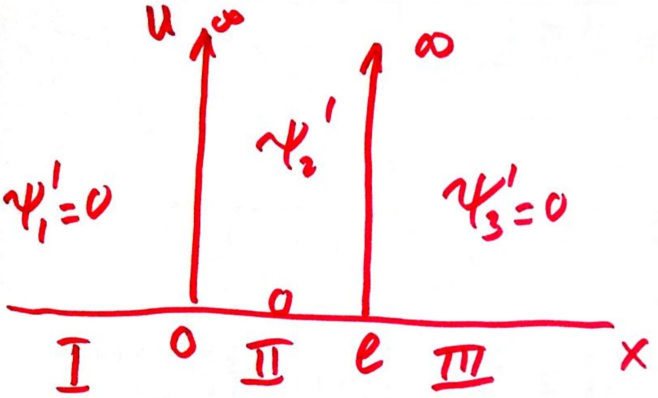
$$\psi'(x) = A e^{\frac{i}{\hbar}\sqrt{2mE}\,x} \qquad e^{-\frac{i}{\hbar}Et}$$

$$\sqrt{2mE} = p$$

$$\psi(x,t) = A e^{-i\frac{E}{\hbar}t + ikx}$$

$$\psi(x,t) = A e^{-\frac{i}{\hbar}(Et - px)}$$

$$|\psi|^2 = \psi \psi^* = A^2$$



$$\nabla^2\psi' + k^2\psi' = 0$$

$$\frac{\partial^2\psi'}{\partial x^2} + \frac{2m}{\hbar^2}(E-U)\psi' = 0$$

$$k^2 = \frac{2m}{\hbar^2}\left(E - U(x)\right)$$

$\psi_1'(0) = \psi_2'(0) = 0 \quad (1)$

$\psi_2'(l) = \psi_3'(l) = 0 \quad (2)$

$$\int_0^l \psi'\psi'^{*} dx = 1$$

$$\psi'(x) = A\sin kx + B\cos kx$$

$$(1) \Rightarrow A\underbrace{\sin(k\cdot 0)}_{=0} + \underbrace{B}_{=0}\cos k\cdot 0 = 0 \Rightarrow \psi'(x) = A\sin kx$$

$$(2) \Rightarrow A\sin kl = 0 \Rightarrow kl = \pm n\pi \qquad n = 1, 2, 3, \ldots$$

$$k = \frac{n\pi}{l}$$

$U(x) = 0$

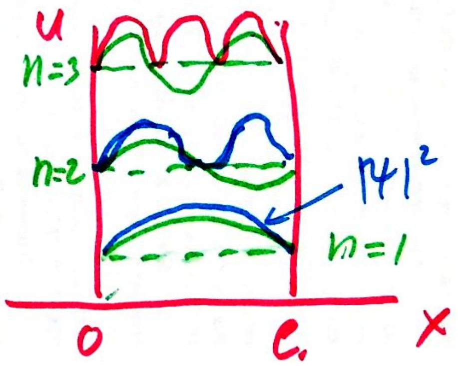
$$\psi'_n(x) = A \sin \frac{n\pi}{\ell} x$$

$$\frac{\partial^2 \psi'}{\partial x^2} + \frac{2mE}{\hbar^2}\psi' = 0 \Rightarrow -\frac{\pi^2 n^2}{\ell^2} + \frac{2mE}{\hbar^2} = 0$$

$$E_n = \frac{\pi^2 \hbar^2}{2m\ell^2} n^2, \quad n = 1, 2, 3$$

$$A^2 \int_0^{\ell} \sin\left(\frac{n\pi}{\ell} x\right) dx = 1 \Rightarrow A = \sqrt{\frac{2}{\ell}}$$

$$\underline{\psi'(x) = \sqrt{\frac{2}{\ell}} \sin \frac{n\pi}{\ell} x}$$



$n = 1$

$n = 2$

$n = 3$

$$\frac{E_{n+1}-E_n}{E_n}=\frac{(n+1)^2-n^2}{n^2}=\frac{2n+1}{n^2}\approx\frac{2}{n}\rightarrow 0$$