

$$\begin{aligned}
\underline{\sqrt{2\pi}\hat{f}(\xi) \cdot \hat{g}(\xi)} &= \sqrt{2\pi}\hat{f}(\xi) \cdot \hat{g}(\xi) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-i\xi x} f(x) dx \cdot \int_{-\infty}^{\infty} e^{-i\xi y} g(y) dy = \\
&= \frac{1}{\sqrt{2\pi}} \iint_{\mathbb{R}^2} e^{-i\xi(x+y)} f(x) g(y) dx dy = \left| \text{замена: } \begin{matrix} x = s \\ y = t - s \end{matrix} \right| = \\
&= \frac{1}{\sqrt{2\pi}} \iint_{\mathbb{R}^2} e^{-i\xi t} f(s) g(t-s) ds dt = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-i\xi t} dt \int_{-\infty}^{\infty} f(s) g(t-s) ds = \\
&= \underline{\widehat{f * g}(\xi)}
\end{aligned}$$